AN APPLICATION OF THE HEAT TRANSFER PROBLEM TO THE CASE OF A DISPERSION LAYER

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The solution to a certain heat transfer problem in the case of a dispersion layer is used for determining the maximum speed of cutting fibrous materials with a plasma jet. The validity of this approximation is verified experimentally on a few examples.

Prominent among the nonmechanical techniques of cutting cloth are those which use plasma jets for breaking down the fibrous material, particularly a concentrated microplasma jet discharged from a small nozzle (not larger than 1 mm in diameter) under a pressure of several atmospheres. Universally applicable and unmatched in simplicity, the microplasma method yields productivity rates not lower than those of any mechanical method used nowadays for continuous single-layer cutting of cloth. This factor as well as the high quality of cuts readily distinguished this method from other nonmechanical methods of cutting fibrous materials.

Just as with those other nonmechanical methods, however, this one too is limited by a maximum attainable cutting speed, on account of the definite time required to break down a fibrous material with the plasma jet. Usually, an analytical expression for the maximum cutting speed is found in such cases by solving the system of differential equations which accounts also for the penetration of the breakdown front and for the phase transformations at the breakdown boundary. Only thermal breakdown is assumed to occur in the material. At the same time, the plasma produces also an appreciable mechanical effect in the cloth. At practical pressure levels and flow rates of the operating gas, the amount of mechanical energy in a microplasma jet is often of the same order as the amount of thermal energy. As a consequence, the role of the thermal energy in a jet is reduced to only heating the material up to its yield-point temperature, whereupon, without reaching its phase-transformation temperature, the cloth skeleton breaks down under the mechanical force of the jet. In this way, phase transformations do not really play any significant role in the cutting of cloth with a microplasma jet.

On the other hand, the small thickness of a layer as well as its appreciable porosity and aeratability during a single-layer microplasma cut allow us to disregard the penetration of the breakdown front and to assume that the temperature rises uniformly over the entire volume where the jet strikes.

With these assumptions concerning the breakdown mechanism, it will be sufficiently accurate to proceed on the basis of a constant-thickness layer, as is usually done in the analysis of drying and heating processes involving dispersion phases in a material.

The problem is thus formulated mathematically on the following premises.

1) The cloth material breaks down not gradually but instantaneously across its thickness, after it has been heated up to its yield-point temperature across the entire thickness from the point of jet application.

2) Lateral heat losses from the hot segment of a cloth are negligible. Therefore, the dimensions of the broken down cloth segment may be assumed as small as the jet diameter and, at the same time, the problem becomes reducible to a one-dimensional one.

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3) For reasons explained earlier, the transverse thermal conductivity of cloth (along the jet) will also be assumed negligible.

With these stipulations, the problem reduces to the well known equation

$$(1+\varphi)\frac{\partial\theta}{\partial u}+\varphi\frac{\partial^2\theta}{\partial u^2}+\frac{\partial^2\theta}{\partial z\partial u}+\frac{\partial\theta}{\partial z}=0 \tag{1}$$

with the boundary conditions

$$\theta|_{u=0}=0;\quad \left.\frac{\partial\theta}{\partial u}\right|_{u=0}=0;\quad \left.\frac{\partial\theta}{\partial z}\right|_{u=0}=0;\quad \left.\frac{\partial\theta}{\partial u}\right|_{z=0}+\theta|_{z=0}=1. \tag{2}$$

Here $\theta = (t-t_0)/(t_S-t_0)$ denotes the dimensionless temperature, $u = \tau\alpha_V/c_0\gamma_0(1-m)$ denotes the dimensionless time, $z = x\alpha_V/wc_1\gamma_1 m$ is the dimensionless space coordinate, and $\varphi = c_1\gamma_1 m/c_0\gamma_0(1-m)$.

Proceeding as in [2, 3] and taking into consideration actual values of c_0, γ_0, m from [4] and c_1, γ_1 from [5], we estimate the value of φ as being larger than 10^{-3}. Therefore, Eq. (1) can be simplified to

$$\frac{\partial\theta}{\partial u}+\frac{\partial^2\theta}{\partial z\partial u}+\frac{\partial\theta}{\partial z}=0. \tag{3}$$

The solution to Eq. (2) with the boundary conditions (2) is [1, 2]:

$$\theta=\exp(-z)\int_0^u\exp(-\xi)\,I_0\left(2\sqrt{z\xi}\right)d\xi, \tag{4}$$

where $I_0(y)$ is a zeroth-order Bessel function of an imaginary argument.

The problem of determining the maximum cutting speed of a cloth with a microplasma jet reduces now to finding $u = f(z, \theta)$ from expression (4) and then, considering that at the limit

$$\tau=\Phi/v_m, \tag{5}$$

$v_m = \psi(x, t_b)$.

Unfortunately, solution (4) for u cannot be written in explicit form. For this reason, we will estimate the magnitude of the dimensionless groups under the integral sign and try to simplify the solution.

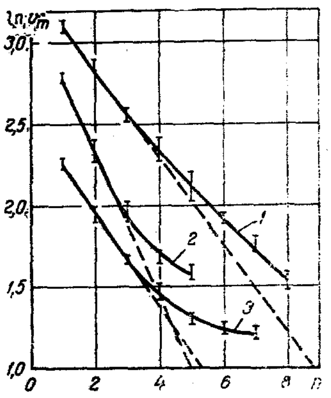
The heat transfer between a gas and a dispersion layer is in most experimental studies dealt with in terms of a surface heat transfer coefficient α_S. The transition from α_S to the volume heat transfer coefficient α_V in this study is easily made by way of simple calculations, if it is assumed that elementary fibers of the yarn are cylinders with an effective mean base radius r. At the same time, the lack of available α_S values for the specific conditions of heat transfer in this problem makes it necessary to resort to averaging. It is convenient to use the universal curve of convective heat transfer in a dispersion layer [6]. For practical cut widths ($\sim$1 mm) and flow rates of plasma generating gas (argon, $\sim$30 cm^3/sec), with a transition to α_V at a temperature not above t_b, this curve yields

$$\alpha_V=\frac{0{,}212\lambda G}{r\mu}(1-m). \tag{6}$$

Considering now the thickness of the cloth, the radius of an elementary yarn fiber [7], the jet velocity ($\sim 10^2$ m/sec, inasmuch as the deceleration of a jet in an air-permeable layer of cloth is negligible), the length of time of jet and cloth interaction (1-5 msec at prevailing cutting speeds of 0.2-0.3 m/sec), and also taking into account the thermophysical properties of both cloth and gas [4, 5], we find that the orders of magnitude of u and z are $\sim$0.1 and $\sim$1.0 respectively.

Based on this estimate, one may expand $I_0(y)$ into a series and integrate expression (4) including only the first few terms of the series. After these operations, with all series terms of higher than second power discarded and the other terms re-arranged, Eq. (4) becomes

$$\theta\exp z=\exp z\left[1-\exp(-u)-u\exp(-u)-\frac{u^2}{2!}\exp(-u)\right]$$
$$+u\exp(-u)+\frac{u^2}{2!}(1+z)\exp(-u). \tag{7}$$



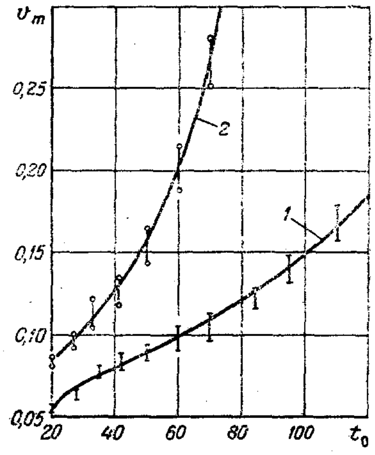


Fig. 1 Fig. 2

Fig. 1. Logarithm of the maximum speed of cutting fibrous material, as a function of the number of cut layers: rayon (1, 2), tracing paper (3).

Fig. 2. Maximum cutting speed v_m (m/sec) as a function of the initial cloth temperature t_0 (°C), under 1.5 atm pressure: cotton paper (t_b = 218°C, α = 0.0271 m/sec, I_d = 3.2 A, V_d = 24 V, Φ = 0.70 mm) 1; nylon (t_b = 104°C, α = 0.0267 m/sec, I_d = 3.5 A, V_d = 20 V, Φ = 0.55 mm) 2.

Expanding now exp(−u) into a series and applying the same rules, we find that the first term on the right-hand side of Eq. (7) vanishes while the whole expression simplifies to

$$\theta \exp z = u + \frac{u^2}{2}(z-1). \tag{8}$$

Equation (8) represents the change in the cloth temperature during the heating process. A cloth can obviously heat up only until its breakdown temperature (yield-point) temperature is reached, whereupon the products of the cloth breakdown are carried away by the operating jet. In order to determine the time required for a cloth to reach its breakdown temperature across the entire layer thickness (assumption 1), therefore, one must let time t become equal to t_b in the expression for the cloth temperature θ. By analogy, one must treat variable x in the dimensionless group z as the cloth thickness.

The desired solution to Eq. (8) is

$$u = \frac{1-\sqrt{1-2\theta(1-z)\exp z}}{1-z}, \tag{9}$$

since, according to the earlier estimate of z, the other root of Eq. (8) yields the absurd condition $u < 0$ when the cloth thickness increases or the jet velocity decreases.

With the aid of Eq. (5), we obtain for the maximum speed of cutting cloth with a microplasma jet

$$v_m = \frac{\Phi\alpha_V}{c_0\gamma_0(1-m)} \cdot \frac{1-z}{1-\sqrt{1-2\theta(1-z)\exp z}}. \tag{10}$$

The accuracy of this expression for v_m has been checked experimentally. In the first series of tests we verified the relation between the maximum cutting speed and the number of cut layers (thickness), using a microplasma jet on several grades of fibrous material. Unfortunately, the lack of data pertaining to actual values of α_V and t_s has prevented us from carrying out a quantitative analysis. Qualitatively, however, the correspondence between Eq. (10) and the experimental relation referred to is easily explainable. For this, we note that at the lowest temperatures t_s = 2000-3000°K and actual breakdown times t_b [7] the values of θ in this problem do not exceed 10^{-1}. Without a significant error, therefore, we may simplify Eq. (10) further for a better overview. Expanding the root of the denominator into a series and discarding all terms of higher than second power, we transform (10) into

$$v_m = \frac{\Phi\alpha_V}{c_0\gamma_0(1-m)\theta} \cdot \frac{\exp(-z)}{1-0.25\theta(z-1)\exp z}, \quad (11)$$

i.e., the maximum speed of cutting cloth with a microplasma jet should decrease almost exponentially as the cloth thickness increases. The actual trend of this relation is determined by the correction terms in the denominator of (11). Test curves of $v_m = \psi(x)$ are shown in Fig. 1 plotted to a semilogarithmic scale. It is easy to see that the first three points of each curve lie on a straight line, within test deviations. The subsequent test points depart from that straight line, owing to the increasing effect of the correction term. The deviation becomes greater as the cloth thickness is increased by adding layers (as x is increased) and as the material density increases (which is reflected in a decreasing w). For sufficiently thin material, according to the initial ranges of the curves in Fig. 1, the maximum speed of cutting a fibrous material with a microplasma jet is an almost exponential function of the layer thickness.

The second semiquantitative method of checking the accuracy of formula (10) was based on following considerations. For two tests performed on one cloth with a plasmatron operating under constant conditions but at very different initial temperatures of the medium t_{01} and t_{02}, the maximum cutting speeds should also have been different. In accordance with that, Eq. (8) was converted into the system

$$\left.\begin{aligned} 2u_1 + u_1^2(1-z) &= 2\theta_1 \exp z \\ 2u_2 + u_2^2(1-z) &= 2\theta_2 \exp z \end{aligned}\right\}, \quad (12)$$

with θ_1 and θ_2 denoting the dimensionless temperature parameters in t_{01} and t_{02} respectively, and u_1, u_2 denoting the process times corresponding to these temperatures.

Dividing one of Eqs. (12) by the other and introducing the cutting speed v_m, we write

$$\frac{\dfrac{1}{v_{m1}} - \dfrac{a}{v_{m1}^2}}{\dfrac{1}{v_{m2}} - \dfrac{a}{v_{m2}^2}} \approx \frac{t_b - t_{01}}{t_b - t_{02}}, \quad (13)$$

where $a = \Phi\alpha_V(1-z)/2c_0\gamma_0(1-m)$ and the sign of approximate equality indicates that t_{01} and t_{02} are negligible relative to t_s, which in our estimate of θ yields an error not greater than 6-7%.

The tests were performed inside a special thermostat. As materials for the specimens we choose two grades of cloth: cotton paper and nylon, having different breakdown temperatures but a similarly low affinity to atmospheric moisture [7]. The latter factor was crucial for reducing the effect of moisture bonded to fibers on the initial range of the test curve.

Since some parameters could be evaluated only qualitatively, for reasons explained earlier, hence the coefficient o for each curve (13) and the breakdown temperature in each case were calculated on the basis of three test points. The resulting $v_m = f(t_0)$ curves are shown in Fig. 2 together with the test points. The close agreement between the curves and the test data indicates that expression (10) is correct.

The evaluated curves in Fig. 2 will also be useful for determining the breakdown temperature of a material cut with a plasma jet. Not surprisingly, the test values for t_b are somewhat lower than those tabulated in [7]. This agrees with our assumption that cloth breaks down at a temperature below its phase-transformation temperature.

The difference in breakdown temperatures due to the consistency of cloths is reflected in the different slopes of the $v_m = f(t_0)$ curves for cotton paper and nylon. The bending of the curve for cotton paper in Fig. 2 within the initial range may be attributed to the fact that higher-power terms in the v_m expression have been disregarded in the series expansion of (4). In (13) all power of the maximum cutting speed appear as reciprocals and, therefore, the contribution of higher-power terms should increase as that speed decreases. For nylon, on the other hand, the slope of the curve is larger and the effect of higher-power terms in v_m is almost negligible.

NOTATION

t is the temperature of cloth skeleton during heating;
t_0 is the initial temperature of cloth skeleton and of gas through pores;
t_s is the temperature of heat source (mean-mass temperature of plasma jet);

t_b is the breakdown temperature of fibrous material;

α_V, α_S is the volume and surface heat transfer coefficients respectively;

τ is the time;

x is the space coordinate;

m is the porosity of cloth layer;

w is the velocity of jet;

c_0, c_1 is the specific heat of cloth skeleton and of plasma generating gas respectively;

γ_0, γ_1 is the specific gravity of cloth skeleton and of plasma generating gas respectively;

Φ is the transverse dimension of the breakdown zone in a fibrous material (equal to the plasma jet diameter, to the first approximation);

v_m is the maximum cutting speed of plasma jet;

r is the equivalent radius of an elementary yarn fiber;

λ, μ is the thermal conductivity and dynamic viscosity of plasma generating gas;

G is the weight-rate of gas flow per section area;

I_d is the arc-discharge current;

V_d is the arc-discharge voltage.

LITERATURE CITED

1. G. D. Rabinovich, Inzh.-Fiz. Zh., 3, No. 4 (1960).
2. G. D. Rabinovich, Theory of Thermal Engineering Calculations on Recuperative Heat Exchangers [in Russian], Izd. Akad. Nauk BSSR, Minsk (1963).
3. A. I. Murav'ev, Study of Heat and Mass Transfer in Engineering Processes and Apparatus [in Russian], Izd. Nauka i Tekhnika, Minsk (1966).
4. P. A. Kolesnikov, Thermal Protective Properties of Garments [in Russian], Legkaya Industriya, Moscow (1965).
5. N. B. Vargaftik, Handbook of Thermophysical Properties of Gases and Liquids [in Russian], Fizmatgiz, Moscow (1963).
6. A. F. Chudnovskii, Heat Transfer in Dispersion Media [in Russian], GITTL, Moscow (1954).
7. G. N. Kukin and A. N. Solov'ev, Textile Materials Science [in Russian], Gostekhizdat, Moscow (1961).
8. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], Fizmatgiz, Moscow (1962).